Object Oriented Modelling & Design of an Interference Radition System

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Contents

1 Abstract: Amplitude, Phase & Frequency Controlled Lobe Rotation

Problem: Given the coordinate systems of the antenna & of the observer at rest in the coordinate system of Earth.

Objective: Rotate the main lobe, such, that it illuminates the receiver.

Assuming all the radiators of equal current moment, for the equal geometry $\underline{\xi}$ the declination of lobes is maximized for maximal difference between respective terms of the expansion of $\mathrm{F}_{\theta\phi}$ (eqn. 27)

$$\cos k\hat{n}\cdot(\underline{\xi} - \underline{\xi}') \cos k\hat{n}\cdot(\delta\underline{\xi} - \delta\underline{\xi}') - \sin k\hat{n}\cdot(\underline{\xi} - \underline{\xi}') \sin k\hat{n}\cdot(\delta\underline{\xi} - \delta\underline{\xi}') \tag{1}$$

determined by $\delta\underline{\xi}$. Maximal absolute rotation value is expected for $\mathrm{k}\ \hat{\mathrm{n}}\cdot(\delta\underline{\xi} - \delta\underline{\xi}') = \mathrm{m}\frac{\pi}{4}$, $\mathrm{m} \in \mathbf{Z}$.

Let all the radiators be fed by equal current with no mutual phase shift. Under these circumstances the system radiates the structure of the highest symmetry, i.e., at given directions there may exist overlaps of more than a single lobe (state). Define the cell (class of radiators) by choosing the range of polar angles & of azimuths. Define the phase shift for members of this class respective elements belonging to the rest set. The phase shift breaks the symmetry of the radiation distribution by **rotating** different states by different elongations. Additionally increase the intensity of currents powering the spirals belonging to the same cell: the given lobes are additionally **enhanced** respective the others.

Low **directional efficiency** (cell to set difference ratio) corresponds to high waist of power (eqn. (9)). But the objective can be fulfilled.

2 Solid Angular Distribution

is the normalized absolute square of the sum of fields measured at $\mathbf{x} \equiv [\mathrm{r}\,\Omega]$ $\equiv [r\,\theta\,\phi]$. Assumed is the Gaussian units' system; c, light velocity. Let $\Omega_{\underline{\xi}} \equiv [\theta_{\underline{\xi}}, \phi_{\underline{\xi}}]$, radiator coordinates in the global spherical system.

Let $\hat{n}$, unit vector in the direction of the observer $\mathbf{x}$, i.e. radius vector of the unit sphere. Let $\underline{\xi}$, the space of sources. Let $\mathrm{k}\hat{n}\cdot\delta\underline{\xi}$, the phase of the radiator located at $\underline{\xi}$. Let F, form factor. The time-averaged power radiated per unit solid angle Ω by the dipole of intensity m oscillating at frequency c k & measured at $\mathbf{x} = \mathrm{r}\hat{n}$ [2]

$$\frac{dP_{\mathbf{m}}}{d\Omega} = \frac{c}{8\pi}[r^2\hat{n}\cdot\mathbf{E}\mathrm{x}\mathbf{B}^*] = \frac{\mathrm{c}}{8\pi}\mathrm{k}^4\mathrm{F}_{\mathbf{m}} \qquad \mathrm{F}_{\mathbf{m}} = |\hat{\mathrm{n}}\mathrm{x}\mathbf{m}\mathrm{x}\hat{\mathrm{n}}|^2 \tag{2}$$

The form factor of the set

$$\begin{aligned} F = |\textstyle\sum_{\underline{\xi}} \hat{n}\mathrm{x}\mathbf{m}_{\underline{\xi}}\mathrm{e}^{\mathrm{ik}\hat{\mathrm{n}}\cdot(\underline{\xi}+\delta\underline{\xi})}\mathrm{x}\mathbf{m}_{\underline{\xi}}|^{\mathbf{2}} = \textstyle\sum_{\underline{\xi}} |\hat{\mathbf{n}}\mathrm{x}\mathbf{m}_{\underline{\xi}}\mathrm{x}\hat{\mathrm{n}}|^2 + \\ + \textstyle\sum_{k\in\mathbf{R}}\sum_{\underline{\xi'}\neq\underline{\xi}}[|\hat{n}\mathrm{x}\mathbf{m}_{\underline{\xi}}\hat{\mathrm{n}}||\hat{\mathrm{n}}\mathrm{x}\mathbf{m}_{\underline{\xi'}}\hat{\mathrm{n}}|\mathrm{e}^{\mathrm{ik}\hat{\mathrm{n}}\cdot(\underline{\xi}+\delta\underline{\xi})}\mathrm{e}^{-\mathrm{ik}\hat{\mathrm{n}}\cdot(\underline{\xi'}+\delta\underline{\xi'})}] \end{aligned} \tag{3}$$

Define the contribution radiated off the front direction, i.e. $\theta \neq 0, \phi \neq 0$, as

$$\begin{aligned} &F_{\theta\phi} = \\ &= \textstyle\sum_{k\in\mathbf{R}}\sum_{\underline{\xi}}[\mathbf{m}_{\underline{\xi}} - \hat{\mathrm{n}}(\hat{\mathrm{n}}\cdot\mathbf{m}_{\underline{\xi}}))(\mathbf{m}_{\underline{\xi'}} - \hat{\mathrm{n}}(\hat{\mathrm{n}}\cdot\mathbf{m}_{\underline{\xi'}}))]\,e^{ik\hat{n}\cdot(\underline{\xi}+\delta\underline{\xi}-\underline{\xi'}-\delta\underline{\xi'})} = \\ &= 2\textstyle\sum\sum_{\underline{\xi'}\neq\underline{\xi}}^{k\geq 0} m_{\underline{\xi'}}m_{\underline{\xi}}(\cos\gamma_{\underline{\xi'}\underline{\xi}} - \cos\theta_{\underline{\xi}}\cos\theta_{\underline{\xi}})\cos\mathbf{k}\cdot(\underline{\xi} + \delta\underline{\xi} - \underline{\xi'} - \delta\underline{\xi'}) \end{aligned} \tag{4}$$

The front maximum is per definition the fraction of the power radiated towards $\theta = 0$:

$$F_{00} = \textstyle\sum_{\underline{\xi}} m_{\underline{\xi}}^2 \sin^2\theta_{\underline{\xi}} \tag{5}$$

2^{nd} index has been deliberately set to zero although in the zenith the azimuth is not defined.

The substitution of (5) & (4) into (11) renders

$$F = F_{00}+$$

$$+2\sum\sum_{\underline{\xi}'\neq\underline{\xi}}^{k\geq 0} m_{\underline{\xi}'}m_{\underline{\xi}}[\cos\theta_{\underline{\xi}}\cos\theta_{\underline{\xi}'}+\sin\theta_{\underline{\xi}}\sin\theta_{\underline{\xi}'}\cos(\phi_{\underline{\xi}}-\phi_{\underline{\xi}'})- \tag{6}$$

$$-\cos\theta_{\underline{\xi}}\cos\theta_{\underline{\xi}'}]\cos\mathbf{k}\cdot(\underline{\xi}+\delta\underline{\xi}-\underline{\xi}'-\delta\underline{\xi}')$$

The zenithal peak is merely intensities' $\mathrm{m}_{\underline{\xi}}$ dependent, while the power radiated towards the off-zenithal directions also depends on the frequency c k & on the elementary phases $\delta_{\underline{\xi}} \equiv \mathbf{k}\cdot\delta\underline{\xi}$, where $\delta\underline{\xi}$, the parameter of the dimension of the length.

$$F = F_{00} + F_{\theta\phi} = \sum_{\underline{\xi}}\{m_{\underline{\xi}}^2\sin^2\theta_{\underline{\xi}}+$$

$$+2\sum_{\underline{\xi}'\neq\underline{\xi}}^{k\geq 0} m_{\underline{\xi}'}m_{\underline{\xi}}\sin\theta_{\underline{\xi}}\sin\theta_{\underline{\xi}'}\cos(\phi_{\underline{\xi}}-\phi_{\underline{\xi}'})\cos k\hat{n}\cdot(\underline{\xi}+\delta\underline{\xi}-\underline{\xi}'-\delta\underline{\xi}')\} \tag{7}$$

The power radiated towards the global direction $\Omega \equiv [\theta\ \phi]$ is the sum of the zenithal maximum $(\frac{dP}{d\Omega})_{00}$ & of the off-zenithal term:

$$\frac{dP}{d\Omega} = (\frac{dP}{d\Omega})_{00} + (\frac{dP}{d\Omega})_{\theta\phi} = \frac{c}{8\pi}k^4(F_{00}+F_{\theta\phi}) \tag{8}$$

Problem: Given the orientations of the moments defined in their elementary coordinate systems $[\theta_{\underline{\xi}}\ \phi_{\underline{\xi}}]$. Determine **frequencies** ω = c k, **phases** of the radiators $\mathbf{k}\cdot\delta\underline{\xi}$ & **amplitudes** $\sqrt{\frac{c}{8\pi}}\ \mathrm{k}^2\mathrm{m}_{\underline{\xi}}$, such, that for $\triangle\theta \to 0$ & $\triangle\phi \to 0 \Rightarrow |\epsilon| \to 0$.

$$I_{\theta\phi} =^{def} \frac{\int_{\phi-\frac{1}{2}\triangle\phi}^{\phi+\frac{1}{2}\triangle\phi} d\phi' \int_{\theta-\frac{1}{2}\triangle\theta}^{\theta+\frac{1}{2}\triangle\theta}\sin\theta' d\theta' \frac{dP}{d\Omega}}{\int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \frac{dP}{d\Omega}} = 1 - |\epsilon| \tag{9}$$

Because the number of the radiators $\mathrm{K}(\{\underline{\xi}\}) \gg 1$, the solution may be rendered by trial & error.

2.1 Accomplishment

The fields interfere at the position $\mathbf{x} \equiv \mathrm{r}\hat{\mathrm{n}}$, $\frac{\mathbf{x}}{|\mathbf{x}|} \equiv \hat{\mathrm{k}} \equiv \hat{\mathrm{n}}$ of the observer. In case that all the elements radiate in phase, i.e $\delta\underline{\xi} \equiv \mathbf{0}\ \forall\underline{\xi}$ the phase factors $\psi_{\underline{\xi}\underline{\xi}'}(\mathrm{k}) = \cos\mathbf{k}\cdot(\underline{\xi}+\delta\underline{\xi}-\underline{\xi}'-\delta\underline{\xi}')$ are merely frequency & geometry dependent.

By feeding the radiators in the direction (θ, ϕ) which should be powerfully illuminated by more intense current, we can increase the amplitudes contributing to the sum $F_{\theta\phi}$. Conversely, the intensities of the radiators at global directions approximately equal to the polar angle $\theta = 0$ should be fed by weaker currents in order to depreciate F_{00} respective the required $F_{\theta\phi}$.

The lobe direction changes by varying the frequency ω = c k & the phases $\psi_{\underline{\xi}} = \mathrm{k}\hat{\mathbf{n}}{\cdot}\delta\underline{\xi}$ [1].

3 The Spiral Radiating Element

Consider the two-arm Archimedian spiral with the arm length a_0 & the spiral constant $a_\zeta =^{def}$

$$\frac{\xi_1}{a_0} = (1 - a_\zeta\zeta)cos\zeta \qquad \frac{\xi_2}{a_0} = (1 - a_\zeta\zeta)sin\zeta \tag{10}$$

In case that the radial coordinate is a very slowly monotonously decreasing function of azimuth in the cylindric $[\rho, \zeta, \xi_3]$ element system the spiral can be approximately treated as the superposition of $n = \frac{a}{a_\zeta}$ pseudocircular (almost exclusively azimuthal) current density $\mathbf{J}_\zeta$ loops of the vanishing radial $J_\rho \to 0$ & no axial $J_{\xi_3} \equiv 0$ component (11). The element radiates the dipole pattern but uses just the lowest term (modified spherical wave) of the multipole expansion of the fields as the form function to evaluate the relation for the solid angular power distribution radiated by R identical elements. Owing to slow convergence of the expansion after spherical Bessel functions the author offers the following analytic derivation (E. Lukacevic, 2002). The element characteristic computed by the designer of the membrane carried system of spiral radiators conforms with the eqn. (27) for the oscillating (ω) magnetic dipole (22) induced by the (in his case approximately) circular current loop.

Consider just a single loop of an infinitely thin wire carrying the intensity I $e^{-i\omega t}$ & lieing in the $x_3 = 0$ plane of the global spherical system [r, θ, ϕ]. $\mathbf{J} = \mathbf{J}_\phi e^{-\mathrm{i}\omega \mathrm{t}}$ [2]. Calculate the magnetic potential in the plane $\phi = 0$. Since J_1 is antisymmetric about $\phi = 0$ the vector potential is induced merely by the component J_2. Notice that the Archimedian spiral shaped wire of vanishing cross section is applied onto the **perfectly insulating surface** of the carrier. It follows that the **source** is physical which means that the current density is no mathematical fiction.

$$\frac{\mathbf{J(x)}}{J_\phi} = -\hat{x_1} sin\zeta + \hat{x_2} cos\zeta \tag{11}$$

Denote by $\mathbf{x} \equiv$ [r, θ, ϕ] the observer system, $\mathbf{x'} \equiv \underline{\xi} \equiv [\xi, \beta, \zeta]$ the system of the source $\mathbf{J}$.

$$|\mathbf{x} - \underline{\xi}|^2 = r^2 - \xi^2 - 2r\xi(cos\theta cos\beta + sin\theta sin\beta cos\zeta) \tag{12}$$

The induced vector potential $\mathbf{A}(\mathbf{x}, t) = \mathbf{A}(\mathbf{x})e^{-i\omega t}$ oscillates with the frequency of the source $\mathbf{J}$ (eqns. (4), (6)). Represent the delta function in spherical coordinates.

$$J_\phi(\mathbf{x}) = I\delta(cos\beta)\frac{\delta(\xi - a)}{a} \tag{13}$$

where $0 < a < a_0$, the radius of the (single) spiral loop considered. From the symmetry of the current (11) $\mathbf{J}(\mathbf{x}) \sim \hat{x_2}$ it follows that
$\mathbf{A}(\mathbf{x}) = A_\phi\ \hat{x_2}$. Introduce (13) into (6)

$$\begin{aligned} A_\phi(r, \theta) &= \frac{I}{ca}\int \xi^2 d\xi d\Omega' \frac{cos\zeta\delta(cos\beta)\delta(\xi - a)}{|\mathbf{x} - \underline{\xi}|} = \\ &= \frac{Ia}{c}\int_0^{2\pi} \frac{cos\zeta d\zeta}{\sqrt{a^2 + \xi^2 - 2a\xi sin\theta cos\zeta}} \end{aligned} \tag{14}$$

where $d\Omega' = sin\beta d\beta d\zeta$, solid angle element of the radiator space. Define the argument k of the complete elliptic integrals K(k), E(k).

$$\begin{aligned} k^2 &= \frac{4ar\ sin\theta}{a^2 + r^2 + 2ar\ sin\theta} \\ A_\phi(r, \theta) &= \frac{4Ia}{c\sqrt{a^2 + r^2 + 2ar\sin\theta}}\frac{(2 - k^2)K(k) - 2E(k)}{k^2} \end{aligned} \tag{15}$$

From the direction of the magnetic induction ∇ x $\mathbf{A} \sim \hat{\mathbf{\Omega}}$ **x** $\mathbf{A}$ & of the electric field curling about it ∇ x $\mathbf{E} + \frac{1}{c}\dot{\mathbf{B}} = 0$; $\mathbf{E} \sim \hat{\Omega}$ x $\mathbf{B}$ we notice that for the vanishing spiral constant a_ϕ (closed current loop) the field is linearly polarized because in the far zone r $\gg$ a it holds for the magnitude of the components $\frac{B_\theta}{B_r} = \frac{1}{2}\tan\theta$ whose phases are equal. For small k^2, corresponding to the near zone a $\gg$ r, our far (radiation) zone a $\ll$ r & the observation direction normal to the plane of the spiral $\theta \ll 1$ (14) reduces to

$$
\begin{aligned}
&lim_{k^2\to 0}\frac{(2-k^2)K(k)-2E(k)}{k^2} = \frac{\pi}{16}k^2 \\
&lim_{k^2\to 0}A_\phi(r,\theta) = \frac{\pi}{c}Ia^2\frac{r\ sin\theta}{(a^2+r^2+2ar\ sin\theta)^{\frac{3}{2}}}
\end{aligned}
\tag{16}
$$

The magnetic induction is oscillating normally to $\hat{\phi}$.

$$
\begin{aligned}
&lim_{k^2\to 0}B_r = \frac{1}{r\ sin\theta}\frac{\partial}{\partial\theta}(sin\theta\ A_\phi) \\
&lim_{k^2\to 0}B_\theta = -\frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) \qquad B_\phi = 0
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&\lim_{k^2\to 0} B_\theta = \frac{\pi}{c}Ia^2\frac{-ar\sin^2\theta - r^2\sin\theta + 2a^2\sin\theta}{(a^2+r^2+2ar\sin\theta)^{\frac{5}{2}}} \\
&\lim_{k^2\to 0} B_r = \frac{\pi}{c}Ia^2\frac{ar\sin\theta\cos\theta + 2(a^2+r^2)\cos\theta}{(a^2+r^2+2ar\sin\theta)^{\frac{5}{2}}}
\end{aligned}
\tag{18}
$$

In the radiation zone a $\ll$ r the magnetic induction

$$
\begin{aligned}
&\lim_{k^2\to 0\&\frac{a}{r}\to 0} B_r = 2\frac{\pi}{c}Ia^2\frac{cos\theta}{r^3} \qquad \lim_{k^2\to 0\&\frac{a}{r}\to 0} B_\theta = \frac{\pi}{c}Ia^2\frac{sin\theta}{r^3} \\
&\lim_{k^2\to 0\&\frac{a}{r}\to 0} \frac{\mathbf{B(x},t)}{B_r}|_{\phi=0} \approx (\hat{r} + \frac{\hat{\theta}}{2}\tan\theta)e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}
\end{aligned}
\tag{19}
$$

(**no** phase difference between $B_r, B_\theta \in \mathbf{R}$) & the electric field *for the* **single** *loop of radius a* are **linearly** polarized. The phases $\mathbf{k}\cdot\mathbf{x} - \omega$t are equal & therefore the polarization is linear notices that the field tends to be linearly polarized off the spiral axes. The magnetic vector is oscillating in the plane ϕ = const along the direction $\tan^{-1}2$ (19).

Superposition of Moments observes the nearly circularly polarized field which means that for the (counter-)clockwise rotation (left: $\psi = \frac{\pi}{2}$, right: $\psi = -\frac{\pi}{2}$ polarization corresponding to the positive & to the negative helicity of the wave, respectively) of the field vector (21) induced by **all** the coils (20)

$B_r, B_\theta, E_\phi \in \mathbf{R} \qquad \mathbf{E} \approx \hat{\mathbf{\Omega}}\mathrm{x}\mathbf{B}$

$$\lim_{k^2 \to 0 \& \frac{a}{r} \to 0} \mathrm{m}(\psi_1, .., \psi_\mathrm{n}, \mathrm{t}, \mathbf{x}) = \frac{\mathrm{I}}{\mathrm{c}} \pi \mathrm{a}^2 \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{e}^{\mathrm{i}\psi_\mathrm{j}} \mathrm{e}^{\mathrm{i}\mathbf{k} \cdot \mathbf{x} - i\omega t} \tag{20}$$

$$E_r = -\frac{m}{r^3} \sin\theta \qquad E_\theta = -\frac{2m}{r^3} \cos\theta \qquad E_\phi = \frac{m}{r^3}(\sin\theta - 2\cos\theta)$$

there holds $|\psi| = \frac{\pi}{2}$.

$$E = \frac{1}{r^3} \sqrt{E_r^2 + E_\theta^2 + E_\phi^2} \sim \frac{1}{r^3} \sqrt{1 + 3\cos^2\theta - \sin 2\theta}$$

$$\mathbf{E} \cdot \hat{\rho} \sim \frac{\cos\theta}{\mathrm{r}^3} \sqrt{1 + 3\cos^2\theta - \sin 2\theta} \tag{21}$$

$$\mathbf{E} \cdot \hat{\xi_3} \sim \frac{\sin\theta}{\mathrm{r}^3} \sqrt{1 + 3\cos^2\theta - \sin 2\theta}$$

For the spiral element (§ 4.1) designed by coiling n tight loops $\mathrm{a}_\zeta \to 0$; $\mathbf{m}$ has to be substituted by

$$\mathbf{m}(\underline{\xi}, t, \psi) = \pi \frac{I}{c} a_0^2 \hat{\xi_3} e^{-i\omega t} \sum_{j=n}^{1} (1 - \frac{j}{n})^2 e^{i\psi_j} \tag{22}$$

3.1 Single-Arm Archimedian Spiral Element

No two-arm Archimedian spiral can be considered, because manufacturing would be too advanced. We must coil the 0.3 mm diameter silver coated copper wire manually. The equation of the single-arm spiral reads in its local system, $\beta = \frac{\pi}{2}$ = const, polar angle defining the equatorial plane:

$$\frac{\triangle\xi}{\xi} = -a_\zeta \triangle\zeta$$

$$lim_{\Xi \to 0} [ln\xi]_a^\Xi = ln\, C - a_\zeta lim_{\Phi \to \infty} [\zeta]_0^\Phi \tag{23}$$

$\mathrm{a}_\zeta =^{def} \frac{\partial\xi}{\partial\zeta} > 0$, spiral constant. a, maximal value of the radial coordinate. Determine the integration constant.

$$ln\, C = -ln\, a \tag{24}$$

Subsequently insert C into (23).

$$ln\frac{\xi}{a} = -a_\zeta \zeta \Rightarrow \xi(\zeta)_{\beta=\frac{\pi}{2}} = a\, e^{-a_\zeta \zeta} \tag{25}$$

Interrupt after n $\in$ **N** pseudoconcentric coils: $\zeta = 2n\pi$, typically n = 5. Manufacturing the mask for 100 open pseudocircular coils, i.e. cutting the channel in the bulk of plastic would require too much effort.

4 Implementation

Rewrite eqns. (11) & (4):

$$F_{00}(\mathbf{x}) = \sum_{\underline{\xi}} \mathrm{m}^2_{\underline{\xi}}[\hat{\mathrm{n}}_{\underline{\xi}} - (\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}_{\underline{\xi}})\hat{\mathbf{n}}]\cdot[\hat{\mathbf{n}}_{\underline{\xi}} - (\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}_{\underline{\xi}})\hat{\mathbf{n}}] \tag{26}$$

$$F_{\theta\phi}(\mathbf{x}) = $$
$$= 2\sum^{k\geq 0}_{\underline{\xi}'\neq\underline{\xi}} m_{\underline{\xi}'} m_{\underline{\xi}}[\hat{n_{\underline{\xi}'}}\cdot\hat{n_{\underline{\xi}}} - (\hat{n}\cdot\hat{n_{\underline{\xi}'}})(\hat{n}\cdot\hat{n_{\underline{\xi}}})] \cos k\hat{n}\cdot(\underline{\xi} + \delta\underline{\xi} - \underline{\xi}' - \delta\underline{\xi}') \tag{27}$$

where obviously $\hat{\mathbf{n}} \equiv \hat{\mathbf{n}}_{\mathbf{x}}$, coordinates of the unit vector pointing outwardly towards the observer located at **x**.

$$\hat{n_{\underline{\xi}}} = \frac{\nabla \mathrm{E}}{|\nabla \mathrm{E}|} \tag{28}$$

is the unit normal pointing outwardly from the surface E(**x**) carrying the radiator at $\underline{\xi}$. In the Cartesian radiator system $\hat{n_{\underline{\xi}}} = [0\ 0\ 1]$ posesses just this representation & it must be transformed into the global system before contracting it by $\hat{n}$ respective $\hat{n_{\underline{\xi}'}}$.

For the spiral element designed by coiling n tight loops of (therefore) nearly equal radii a carrying current intensity I [statampère] = $\frac{\mathrm{c}}{10}$ I [ampère]

$$m_{\underline{\xi}} = \pi \frac{I}{c} n a^2 \tag{29}$$

4.1 Occupation of the Columns

Table 1: layer[l].row[i]→contra (Linux g++)

j	plate.layer[l].row[i].contra[j]	Explanation
	Geometry	**l = 0**
0	$\phi_{p\gamma} = \phi_\gamma^{pc} \pm \frac{1}{2^j} \triangle\Phi_\gamma$	cylinder [cm]
1	$\theta_p = \theta_{pc} \pm \frac{1}{2^j} \triangle\Theta_\gamma$ ellipsoid	$\mathrm{x_{3p}} = \pm\frac{\mathrm{M}}{2^{\mathrm{j}}}\mathrm{a}_1$
2	ξ_3	
3	$\xi_3 = \mathrm{e}_{23} \sin\theta_{\mathrm{p0}} \sin\phi_{\mathrm{p0}} + \mathrm{e}_{23} \sin\theta_{\mathrm{p}} \sin\phi_{\mathrm{p}}$	
4	$\xi_1 = \cos\theta_p^\gamma$	global Cartesian
5	$\xi_2 = \mathrm{e}_{\alpha\gamma} \sin\theta_p^\gamma \cos\phi_p^\gamma$	
6	radial $\xi = \sqrt{x_1^2 + x_2^2 + x_3^2}$	global polar coords
7	$\theta_{\underline{\xi}} = \cos^{-1} \frac{\mathrm{x}_3}{\sqrt{\mathrm{x}_1^2+\mathrm{x}_2^2+\mathrm{x}_3^2}}$	polar
8	$\phi_{\underline{\xi}} = \cos^{-1} \frac{\mathrm{x}_1}{\sqrt{\mathrm{x}_1^2+\mathrm{x}_2^2}}$	azimuthal
	Dipoles	**l = 1**
0	$(\hat{\mathrm{n}}_{\underline{\xi}})_1$	global Cartesian
1	$(\hat{\mathrm{n}}_{\underline{\xi}})_2$	radiator position
2	$(\hat{\mathrm{n}}_{\underline{\xi}})_3$	plate normal
3	$\mathrm{m}_{\underline{\xi}}$	[esu cm]
4	$\delta_{\underline{\xi}}$	phase difference
5	$\mathrm{k}_{\underline{\xi}}$	wave number
	Observers	**l = 0, 3, 5, 7,...**
9	$\theta_{\mathbf{x}}$	polar coords of the
10	$\phi_{\mathbf{x}}$	observer at **x**
11	x_1	observer Cartesian coords
12	x_2	for distance $\mathrm{r}_{\mathbf{x}} = 1$
13	x_3	
	Intensities or Coils	**l = 2, 4, 6, 8,...**
14	F_1	$\mathrm{F} \sin\theta_{\mathbf{x}} \cos\phi_{\mathbf{x}}$
15	F_2	$\mathrm{F} \sin\theta_{\mathbf{x}} \sin\phi_{\mathbf{x}}$
16	F_3	$\mathrm{F} \cos\theta_{\mathbf{x}}$
14	c_1 out by print_coils &	connect serially
15	c_2 print_Poynting_shape	subsequent radiators
16	c_3 wire	global coords

Table 2: layer[l][i][j] (Visual C++)

j	layer[l][i].contra[j]	Explanation
	Geometry	**l = 0**
0	$\phi_{p\gamma} = \phi_{\gamma}^{pc} \pm \frac{1}{2^j} \triangle\Phi_{\gamma}$	cylinder [cm]
1	$\theta_p = \theta_{pc} \pm \frac{1}{2^j} \triangle\Theta_{\gamma}$ ellipsoid	$\mathrm{x_{3p}} = \pm \frac{\mathrm{M}}{2^j} \mathrm{a}_1$
2	$\xi_1 = \cos\theta_p^{\gamma}$	global Cartesian
3	$\xi_2 = \mathrm{e}_{\alpha\gamma} \sin\theta_p^{\gamma} \cos\phi_p^{\gamma}$	
4	$\xi_3 = \mathrm{e}_{23} \sin\theta_{\mathrm{p0}} \sin\phi_{\mathrm{p0}} + \mathrm{e}_{23} \sin\theta_{\mathrm{p}} \sin\phi_{\mathrm{p}}$	
5	radial $\xi = \sqrt{x_1^2 + x_2^2 + x_3^2}$	global polar coords
6	$\theta_{\underline{\xi}} = \cos^{-1} \frac{\mathrm{x}_3}{\sqrt{\mathrm{x}_1^2+\mathrm{x}_2^2+\mathrm{x}_3^2}}$	polar
7	$\phi_{\underline{\xi}} = \cos^{-1} \frac{\mathrm{x}_1}{\sqrt{\mathrm{x}_1^2+\mathrm{x}_2^2}}$	azimuthal
	Dipoles	**l = 1**
0	$(\hat{\mathrm{n}}_{\underline{\xi}})_1$	global Cartesian
1	$(\hat{\mathrm{n}}_{\underline{\xi}})_2$	radiator position
2	$(\hat{\mathrm{n}}_{\underline{\xi}})_3$	plate normal
3	$\mathrm{m}_{\underline{\xi}}$	[esu cm]
4	$\delta_{\underline{\xi}}$	phase difference
	Observers	**l = 2, 3, 4, 5, 6,...**
0	$\theta_{\mathbf{x}}$	polar coords of the
1	$\phi_{\mathbf{x}}$	observer at $\mathbf{x}$
2	x_1	observer Cartesian coords
3	x_2	for distance $\mathrm{r}_{\mathbf{x}} = 1$
4	x_3	
	Intensities	
5	F_1	$\mathrm{F} \sin\theta_{\mathbf{x}} \cos\phi_{\mathbf{x}}$
6	F_2	$\mathrm{F} \sin\theta_{\mathbf{x}} \sin\phi_{\mathbf{x}}$
7	F_3	$\mathrm{F} \cos\theta_{\mathbf{x}}$

4.2 Input Options

Table 3: Input Options

*option[1]	Declarator	Storage Class	Variable Assignment & Explanation
o	fout	string	output file/${PBS_JOBNAME}.
n	color_table_index > 1	int	
	Plate		
U	polar_pc	double	θ_{pc} vis. ρ_{pc}
V	azimuth_pc	double	ϕ_{pc}
P	log2patches	int	number of patches along the axis of the θ_3^p definition & also along the circumference is 2^P
t	polar_semirange_flag	char	if > 0 both semiranges between both caps; otherwise between the northern cap & the equator
T	polar-/radial_semirange; pseudocone: $\alpha_1 \bigvee \alpha_3$	double	$\frac{1}{2}\triangle\Theta_p$ (ellipsoid or body with n^{th} order radial cross section P_n)/$\frac{1}{2}\triangle R$
F	azimuth_semirange	double	$\frac{1}{2}\triangle\Phi_p$; e.g. $\frac{\pi}{2}$
B	eccentricities[0]; point: -1; cylinder & pseudocone: 0; spheroid: 1; general ellipsoid: 2; paraboloid: 'p' & eccentricities[2] = $r_d > 0$ & paraboloid order $\frac{r}{r_d}$	char*	hyperboloid: eccentricities[0] = 'h', eccentricities[1] = r = hyper(para)boloid_order; P: body of radial cross sections bounded by P_n, n^{th} order curve
p	cylinder_magnification_factor M	double	$a_3 = Ma_1$ (otherwise unused)
a	$a_\alpha \rightarrow e_{\alpha\gamma}$/open surface eccentricity; P4: $e_{\beta\gamma} \equiv e_{12} = \frac{a_1}{a_2}$	double	stores the order of the cross section P_n; spheroid: input value either of a_α or of a_β must be defined = 0
b	$a_\beta \rightarrow e_{\beta\gamma}$/open surface ascent parameter; pseudocone: input values $a_\alpha = a_\beta$ must be equal & > 0	double	if hyperboloid_order < 0 in cm; P_n: stores the parameter e
C	a_γ; hyper(para)boloid: unused	double	pseudocone: $2\cdot\frac{1}{2}\triangle\Psi = \alpha_{2,4} - \alpha_{1,3}$
c	polars; e.g. 1; if polars < 0, polars → -polars, r → $\frac{1}{r}$	int	generate_observers argument
s	radiator_zoom_factor	double	radiator locus norm magnification factor
	Antenna Cell		
D	$\theta_{\psi c}$ cell_center_polar	double	diff_phase[0]
O	$\triangle\theta_\psi$ cell_polar_semirange	double	diff_phase[1]
Z	$\phi_{\psi c}$ cell_center_azimuth	double	diff_phase[2]
I	$\triangle\phi_\psi$ cell_azimuth_semirange	double	diff_phase[3]
R	$\delta_{\underline{\xi}} = k\hat{\mathbf{n}}\cdot\delta\underline{\xi}$ phase_difference	double	diff_phase[4]
W	current_intensity_magnification_factor		$i_{\underline{\xi}}$ double

*option[1]	Declarator	Storage Class	Variable Assignment & Explanation
f	linear_frequency: if $<$ 0 hyper(para)boloid_order r $\rightarrow$ -r (option B)	double	(e.g. $2{\cdot}10^9$) [s^{-1}]; the program calculates the wave_number [cm^{-1}] always > 0
	Tensorial Point Source		
d	spiral_radius	double	e.g. 0.5 cm (also start for get_global_system_geometry)
i	current_intensity	double	e.g. 10 ampère
	Locus of Observers		
E	observer_locus_zoom_factor: if Poynting_shape_zoom_factor < 0 intensity vector starts at origin	double	radius of the sphere of observers is depreciated/expanded in order to render nice graphics
S	Poynting_shape_zoom_factor > 0: add intensity to the point of measurement; < 0: $[\frac{\text{erg}}{\text{s}}]$	double	multiplies the Poynting vector norm of the method define_characteristic
r	observer_cone_polar_angle	double	polar cap to define the observers on the spherical helix, $0 < \theta < \pi$; e.g. $\frac{\pi}{64}$
m	min_polar	double	of the polar belt, $0 \leq \theta_0 \ll \frac{\pi}{2}$
M	step_polar	double	width of the polar belt, e.g. $\frac{\pi}{64}$
l	min_azimuth; e.g. 0	double	on the observers' locus
L	max_azimuth; e.g. π	double	on the sphere of observers

5 Cylinder on Absolute Units

$$\mathbf{x_0} \equiv [a_1, \phi_{pc}, x_3^{p0}]$$

$$\mathbf{x_1^i} = \mathbf{x_0} + \delta\mathbf{x_1^i} \qquad \delta\mathbf{x_1^i} = [0, \phi_{pc} \underline{+} \tfrac{1}{2} \triangle \Phi, x_3^{p0} \underline{+} \tfrac{1}{2} a_3 \triangle \Phi]$$

$$\mathbf{x_2^i} = \mathbf{x_1} + \delta\mathbf{x_2^i} \qquad \delta\mathbf{x_2^i} = [0, \phi_{pc} \underline{+} \tfrac{1}{2^2} \triangle \Phi, x_3^{p0} \underline{+} \tfrac{a_3}{2^2} \triangle \Phi]$$

$$\text{...........} \qquad i \in \{1,2,3,4\} \tag{30}$$

$$\mathbf{x_j^i} = \mathbf{x_{j-1}} + \delta\mathbf{x_j^i} \qquad \delta\mathbf{x_j^i} = [0, \phi_{pc} \underline{+} \tfrac{1}{2^j} \triangle \Phi, x_3^{p0} \underline{+} \tfrac{a_3}{2^j} \triangle \Phi]$$

$$\text{...........} \qquad j \in \{0,1,..,P-1,P\} \qquad \mathbf{x_{P-1}^i} = \mathbf{x_{P-2}} + \delta\mathbf{x_{P-1}^i}$$

$$\delta\mathbf{x_{P-1}^i} = [0, \phi_{pc} \underline{+} \tfrac{1}{2^{P-1}} \triangle \Phi, x_3^{p0} \underline{+} \tfrac{a_3}{2^{P-1}} \triangle \Phi]$$

$$\mathbf{x_P^i} = \mathbf{x_{P-1}} + \delta\mathbf{x_P^i} \qquad \delta\mathbf{x_P^i} = [0, \phi_{pc} \underline{+} \tfrac{1}{2^P} \triangle \Phi, x_3^{p0} \underline{+} \tfrac{a_3}{2^P} \triangle \Phi]$$

$$x_2 = -x_2^{(p)} \qquad x_3^{(p)} = -x_1 \qquad -a_1 + x_1^{(p)} = x_3 \qquad a = a_1 \tag{31}$$

$$x_{1p}^2 + x_{2p}^2 - a^2 = 2E(\mathbf{x})$$

$$(a + x_3)^2 + x_2^2 - a^2 = 2E(\mathbf{x}) \tag{32}$$

$$(a + x_3)\hat{x_3} + \mathbf{x_2} = \nabla \mathrm{E} = \underline{\xi_3}$$

A vector lieing in the tangential plane $\underline{\xi_3} \cdot \underline{\xi_1}$:

$$\underline{\xi_1} = (a + x_3)\hat{x_2} - \mathbf{x_3} \tag{33}$$

Determine yet another vector lieing in the tangential plane $\underline{\xi_3}$x$\underline{\xi_1}$.

$$\underline{\xi_2} = -[1 + (a + x_3)^2]\hat{x_1} \tag{34}$$

The elements of the unitary transformer into the global system are unit vectors $\hat{\xi^{\alpha}}$

$$\Xi = \begin{pmatrix} \hat{\xi^{1}} \\ \hat{\xi^{2}} \\ \hat{\xi^{3}} \end{pmatrix} = \begin{pmatrix} 0 & \xi^{\hat{1}2} & \xi^{\hat{1}3} \\ -1 & 0 & 0 \\ 0 & \xi^{\hat{3}2} & \xi^{\hat{3}3} \end{pmatrix} \tag{35}$$

References

[1] Karl Rothammel & Alois Krischke: Antennenbuch. 11^{th} Edition. Stuttgart: Franckh-Kosmos, 1995. § 4.4. p. 79; Rs 7200 R845(11); 2E 1608 (11) Stadtmitte

[2] John David Jackson: Classical Electrodynamics, 2^{nd} Edition, *John Wiley* & *Sons, Inc.,* New York, London, Sydney, Toronto (1975).